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Laplace Transforms: Heaviside function

Numeracy Workshop

Geoff Coates
Introduction

*These slides cover the application of Laplace Transforms to Heaviside functions. See the Laplace Transforms workshop if you need to revise this topic first. These slides are not a resource provided by your lecturers in this unit.*
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**Introduction**

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Example: \( f(t) = \begin{cases} 0 & , \ t < 1 \\ 2 & , \ 1 \leq t < 3 \\ t & , \ t \geq 3 \end{cases} \)
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The **Heaviside function** can do this:

\[
H(t) = \begin{cases} 
0 & , 
\text{ } t < 0 \\
1 & , 
\text{ } t \geq 0 
\end{cases}
\]
The Heaviside function

Multiply a function $g(t)$ by $H(t)$ and it will “turn $g(t)$ on” at $t = 0$:

If $g(t) = t^2 + 1$, then $g(t)H(t)$ looks like this:
The Heaviside function

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The Heaviside function

To “turn functions on” at points other than zero, say $a$, we replace $t$ by $t - a$: 

$$H(t - a) = \begin{cases} 
0, & t < a \\
1, & t \geq a 
\end{cases}$$
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H(t - a) = \begin{cases} 
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1 & , \quad t \geq a
\end{cases}
\]

For $t < a$,
\[
H(t - a) - H(t - b) = 0 - 0 = 0.
\]

For $a \leq t < b$,
\[
H(t - a) - H(t - b) = 1 - 0 = 1.
\]

For $t \geq b$,
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For $t < a$, \( H(t - a) - H(t - b) = 0 - 0 = 0 \).

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Back to our example:

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\[ f(t) = 2[H(t - 1) - H(t - 3)] + \\
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f(t) = 2[H(t - 1) - H(t - 3)] + tH(t - 3)
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- turn 3rd branch on at \( t = 3 \)
The Heaviside function

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Laplace transform of the Heaviside function

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Now we need to know something about the Laplace Transforms of Heaviside functions.
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\[ \mathcal{L}[H(t - 1)] = ? \quad \mathcal{L}[H(t - 3)] = ? \quad \mathcal{L}[tH(t - 3)] = ? \]
Laplace transform of the Heaviside function

Theorem 8.27 in the MATH1002 notes says:

\[ \mathcal{L}[f(t - a)H(t - a)] = e^{-as}F(s) \]
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We need the Laplace transform of just a Heaviside function, \( \mathcal{L}[H(t - a)] \) so it makes sense to choose \( f(t) = 1 \). Why?
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\[
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\begin{align*}
\mathcal{L}[H(t - a)] &= e^{-as}F(s) \\
&= e^{-as} \times \frac{1}{s} \\
&= \frac{e^{-as}}{s}
\end{align*}
\]

Now we know that \( \mathcal{L}[H(t - 1)] = \frac{e^{-s}}{s} \) and \( \mathcal{L}[H(t - 3)] = \frac{e^{-3s}}{s} \).
Laplace transform of the Heaviside function

\[ \mathcal{L}[f(t - a)H(t - a)] = e^{-as} F(s) \]

To find \( \mathcal{L}[tH(t - 3)] \),
Laplace transform of the Heaviside function

\[ \mathcal{L}[f(t - a)H(t - a)] = e^{-as}F(s) \]

To find \( \mathcal{L}[tH(t - 3)] \), make \( f(t) = t \) (so \( F(s) = \frac{1}{s^2} \)).
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The theorem uses \( f(t - 3) = t - 3 \) so we need to make an adjustment before we can apply it to \( \mathcal{L}[tH(t - 3)] \):
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\]

\[
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\[
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Laplace transform of the Heaviside function

\[ \mathcal{L}[f(t - a)H(t - a)] = e^{-as}F(s) \]

**Note:** The method we just used is essentially what the MM2 notes does.
Laplace transform of the Heaviside function

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Laplace transform of the Heaviside function

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**Note:** The method we just used is essentially what the MM2 notes does. You might find it intuitively easier to simply declare \( f(t - 3) = t \).

The trouble is that \( f(t) = t + 3 \) and we don’t have the Laplace transform of \( t + 3 \). However,
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\]
Laplace transform of the Heaviside function

\[ \mathcal{L}[f(t - a)H(t - a)] = e^{-as}F(s) \]

**Note:** The method we just used is essentially what the MM2 notes does. You might find it intuitively easier to simply declare \( f(t - 3) = t \).

The trouble is that \( f(t) = t + 3 \) and we don’t have the Laplace transform of \( t + 3 \). However,

\[
F(s) = \mathcal{L}[f(t)] = \mathcal{L}[t + 3] = \mathcal{L}[t] + 3\mathcal{L}[1] = \frac{1}{s^2} + 3 \frac{1}{s}
\]

Using this in the theorem leads to the same answer (with the same amount of work as for the previous method).
Finally, the answer is:

\[ F(s) = 2\mathcal{L}[H(t - 1)] - 2\mathcal{L}[H(t - 3)] + \mathcal{L}[tH(t - 3)] \]
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\[ = \frac{2se^{-s} + se^{-3s} + 2e^{-s}}{s^2} \]
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